# Late time acceleration in Brans Dicke Cosmology

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In this work we have investigated the possibility of having a late time accelerated phase of the universe, suggested by recent supernova observation, in the context of Brans Dicke (BD) theory with a symmetry breaking kind of potential and a matter field. We find that a perfect fluid kind of matter (pressureless and with pressure) cannot support this acceleration but a fluid with dissipative effect can drive this late time acceleration. We have also calculated some cosmological parameters in our model to match with observations.

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## I. INTRODUCTION

A lot of activity has been trigerred by two recent observations [1,2] on the explosion of type Ia Supernovae. These datas favour the existence of a new kind of matter with positive energy density dominant at present universe and is also responsible for the present acceleration of the universe accounted by its negative pressure. This along with the observed location of the first acoustic peak of CMB temperature fluctuation corroborated by the latest BOOMERANG and MAXIMA data [3,4], favours a spatially flat universe whose energy density is dominated by a cosmological constant like term. Obviously the first natural choice to represent such special matter was the cosmological constant $\Lambda$ [5,6]. For a flat matter dominated universe with $\Lambda$ having $\Omega_\Lambda \sim 0.72$ in Einstein gravity best fits the data sets. But the candidature of $\Lambda$ as the constituent of the major energy density is troubled by the fact that it has an energy scale which is $\sim 10^{-123}$ lower than normal energy scale predicted by the most particle physics models. So to find some alternative candidate for this acceleration a dynamical $\Lambda$ [7] in the form of a scalar field with some self interacting potential [8] is considered whose slowly varying energy denisty mimics an effective cosmological constant. The idea of this candidate, called *quintessence* [7], is borrowed from the inflationary phase of the early universe, with the difference that it evolves at a much lower energy scale. The energy density of this field, though dominant at present epoch, must remain subdominant at very early stages and has to evolve in such a way that it becomes comparable with the matter density $\Omega_m$ now. This type of specific evolution, better known as "*cosmic coincidence*" [9] problem, needs several constraints and fine tuning of parameters for the potential used to model quintessence with minimally coupled scalar field. A new form of quintessence field called "*tracker field*" [10] has been proposed to solve the cosmic coincidence problem. It has an equation of motion with an attractor like solution in a sense that for a wide range of initial condition the equation of motion converges to the same solution.

There are a number of quintessence models which have been put forward and most of which involve minimally coupled scalar field with different potentials dominating over the kinetic energy of the field. A purely exponential potential is one of the widely studied cases [11]. Inspite of the other advantages the energy density is not enough to make up for the missing part. Inverse power law is the other potential ( [8]- [10]) that has been studied extensively for quintessence models, particularly for solving the cosmic coincidence problem. Though the problems are resolved successfully with this potential, the predicted value for the equation of state for the quintessence field, $\gamma_Q$, is not in good agreement with the observed results. In search of proper potentials that would eliminate the problems, new types of potentials, like $V_0[\cos h\lambda\phi - 1]^p$ [14] and $V_0 \sin h(\alpha\sqrt{k_0}\Delta\phi)^\beta$ [6,15] have been considered, which have asymptotic forms like the inverse power law or exponential ones. Different physical considerations have lead to the study of other types of the potentials also [16]. Recently Saini *et al* [17] have reconstructed the potential in context of general relativity and minimally coupled quintessence field from the expression of the luminosity distance $d_L(z)$ as function of redshift obtained from the observational data. However, none of these potentials are entirely free of problems. Hence, there is still a need to identify appropriate potentials to explain current observations [11].

Most of the studies regarding accelerated expansion have been done with a minimally coupled scalar field representing the quintessence. It has been recently shown by Pietro and Demaret [12] that for constant scalar field equation of state, which is a good approximation for a tracker field solutions, the field equations and the conservation equations strongly constrain the scalar field potential, and most of the widely used potential for quintessence, such as inverse power law one, exponential or the cosine form, are incompatible with these constraints. The minimally coupled self interacting models will also be ruled out if the observations predict that the missing component of the energy density obeys an equation of state $p = \gamma\rho$ with $\gamma < -1 (\rho \geq 0)$ , and these sort of equation of state is in reasonable agreement with different observations [13]. Also the inequality $dH^2(z)/dz \geq 3\Omega_{m0}H_0(1+z)^2$ should statisfy for minimally coupled scalar field and its violation will certainly point towards a theory of non Einstein gravity such as scalar tensor theories where the scalar field is non minimally coupled to gravity.

There have been quite a few attempts of treating this problem with the non-minimally coupled scalar fields. Scaling attractor solutions are available in the literature with the exponential [16] and power law [16,18] potentials in non-minimally coupled theories. Faraoni [19] have studied different potentials with a non-minimal coupling term $\psi R\frac{\phi^2}{2}$ for the present acceleration. There have been different approach also for solving the problem in general scalar tensor theory, sometimes called *extended* or *generalised* quintessence, not only because this theory is considered to be the most natural alternative to general relativity, there are other strong motivations [20] also. People like Bertolo *et al* [21], Bertolami *et al* [22], Ritis *et al* [23] have found tracking solution in scalar tensor theories with different types of power law potential. In another work Sen *et al* [24] have found the potential relevant to power law expansion in Brans Dicke cosmology. Like Saini *et al*, Boisseau *et al* [25] have reconstructed the potential from the luminosity-redshift relation available from the obervations in context of scalar tensor theory.

Very recently Mcdonald [30] has investigated the possibilty for modelling a dynamical cosmological constant with a scalar field which has undergone to a very recent phase transition. For these he has considered a standard $\phi^4$ potential for the scalar field with an additional time dependent mass squared term in the potential which has become negative very recently. For this kind of model phase transition occur very recently at redshift $z \leq 1.2$.

In these paper we have investigated whether non minimally coupled self interacting scalar field such as a Brans-Dicke (BD) type scalar field with this kind potential can successfully drive the late time acceleration for the flat universe. In the context of Brans Dicke(BD) theory [29] with a self interacting potential and a matter field, the action is given by

$$\mathcal{S} = \int d^4x\sqrt{-g}[\phi R - \frac{\omega}{\phi}\phi^\alpha\phi_\alpha - V(\phi) + \mathcal{L}_m] \tag{1}$$

(We have chosen the unit $8\pi G_0 = c = 1$.)
For the potential we would consider one that usually mimics a very recent phase transition [30]

$$V(\phi) = \lambda\phi^4 - \mu^2(t)\phi^2 \tag{2}$$

where

$$\mu^2(t) = \bar{\mu_0}^2\left(\frac{a_c}{a}\right)^n = \frac{\mu_0^2}{a^n} \tag{3}$$

$\mu_0^2 = \bar{\mu_0}^2 {a_c}^n$. $\lambda$ is a constant and $n$ is an integer. The time dependent mass squared term with integer $n$ can arise naturally in plausible models and one can have a detailed discussion in [30].

As a matter field we would consider first perfect fluid and then a fluid having negative pressure.An effective negative pressure and hence an acceleration can be achieved by dissipative mechanism modelled commonly by fluid viscosities. It has been proposed recently that the CDM must self interact in order to explain the detailed structure of the galactic halos [27]. This self interaction will naturally create a viscous pressure whose magnitude will depend on the mean free path of the CDM particles. An effective negative pressure in CDM can also be created from cosmic anti friction which is closely related to particle production out of gravitational field [28]. Since the negative pressure can be modelled in two different ways we are not apriori assuming any specific model for this negative pressure.

In this work we find that it is not possible to have an late time accelerated expansion when the CDM is a perfect fluid, but a dissipative CDM fluid in BD cosmology with such a potential like (2) can successfuly drive a late time accelerated expansion. In the next section we treat the field equations and find the solutions in both the cases. We also calculate some cosmological parameters to match the accelerated model with observation. The third section is the concluding section where we have discussed different features of this model.

## II. FIELD EQUATIONS AND SOLUTIONS

The gravitational field equations derived from the action (1) by varying the action with respect to the metric is,

$$\begin{aligned} G_{\mu\nu} = \frac{T_{\mu\nu}}{\phi} + \frac{\omega}{\phi^2}(\phi_\mu\phi_\nu - \frac{1}{2}g_{\mu\nu}\phi_\alpha\phi^\alpha) \\ + \frac{1}{\phi}[\phi_{\mu;\nu} - g_{\mu\nu}\Box\phi] - g_{\mu\nu}\frac{V(\phi)}{2\phi} \end{aligned} \tag{4}$$

where $T_{\mu\nu}$ represents the energy momentum tensor of the matter field. We have assumed the matter content of the universe to be composed of a fluid represented by the energy momentum tensor

$$T_{\mu\nu} = (\rho + P)v_\mu v_\nu + Pg_{\mu\nu}, \tag{5}$$

where $\rho$ and $P$ are the energy density and effective pressure of the fluid respectively and $v_\mu$is the four velocity of the fluid i.e, $v_\mu v^\mu = -1$. The effective pressure of the fluid includes the thermodynamic pressure $p$ and a negative pressure $\pi$, which could arise either because of the viscous effect or due to particle production, i.e,

$$P = p + \pi \tag{6}$$

The wave equation that follows from equation (1), by varying the action with respect to the scalar field $\phi$ is

$$\Box\phi = \frac{T}{2\omega+3} + \frac{1}{2\omega+3}\left(\phi\frac{dV(\phi)}{d\phi} - 2V(\phi)\right) \tag{7}$$

For our choice of potential (2), the field equations (4) and the wave equation (7)becomes

$$3\frac{\dot{R}^2}{R^2} + 3\frac{\dot{R}}{R}\frac{\dot{\phi}}{\phi} - \frac{\omega}{2}\frac{\dot{\phi}^2}{\phi^2} - \frac{\lambda}{2}\phi^3 + \frac{\mu_0^2}{2R^n}\phi = \frac{\rho}{\phi}, \tag{8}$$

$$2\frac{\ddot{R}}{R}+\frac{\dot{R}^2}{R^2}+\frac{\ddot{\phi}}{\phi}+2\frac{\dot{R}}{R}\frac{\dot{\phi}}{\phi}+\frac{\omega}{2}\frac{\dot{\phi}^2}{\phi^2}-\frac{\lambda}{2}\phi^3+\frac{\mu_0^2}{2R^n}\phi=-\frac{p}{\phi} \tag{9}$$

and

$$\ddot{\phi}+3\frac{\dot{R}}{R}\dot{\phi}=\frac{\rho-3p}{2\omega+3}-\frac{1}{2\omega+3}\left[2\lambda\phi^4+\frac{n\mu_0^2\phi^2}{R^n}\frac{\frac{\dot{R}}{R}}{\frac{\dot{\phi}}{\phi}}\right] \tag{10}$$

We have assumed standard Friedman-Robertson-Walker metric with the signature convention $(-,+,+,+)$ and $R$ is the scale factor. We restrict ourselves for spatially flat metric only. We work in Jordan frame. One interesting thing about BD theory in Jordan frame is that the conservation equation holds for matter and scalar field separately. Or in a slightly different way, the Bianchi Identity along with the wave equation (7) gives the matter conservation equation

$$\dot{\rho}+3\frac{\dot{R}}{R}(\rho+p+\pi)=0 \tag{11}$$

We assume both the scale factor and the scalar field evolve as the power function of time

$$R=R_0\left(\frac{t}{t_0}\right)^{\alpha} \quad \text{and} \quad \phi=\phi_0\left(\frac{t}{t_0}\right)^{\beta} \tag{12}$$

where the subscript $0$ refers to the values of the parameters at the present epoch. In order to get accelerated expansion for such a evolution of the universe the deceleration parameter has to be negative, which immediately restricts the parameter $\alpha$ to be greater than 1. For such an expansion the solution for the matter density is

$$\rho=\rho_c t^{\beta-2} \tag{13}$$

where

$$\rho_c=\frac{3\alpha\phi_0}{t_o^{\beta}}\left[\frac{2\alpha+\beta(1+\alpha)-\beta^2(1+\omega)}{2-\beta}\right] \tag{14}$$

First we consider normal perfect fluid with no negative pressure i.e, $\pi=0$ in equation (6). Then, for the power law evolution the themodynamic pressure of the fluid becomes

$$p=p_c t^{\beta-2} \tag{15}$$

where

$$p_c=\frac{(2-\beta-3\alpha)\phi_0}{t_o^{\beta}}\left[\frac{2\alpha+\beta(1+\alpha)-\beta^2(1+\omega)}{2-\beta}\right] \tag{16}$$

Power law solution is consistent with the field equations (8), (9) and (10) only if

$$\beta=-\frac{2}{3} \quad \text{and} \quad \alpha n-\beta=2 \quad \text{i.e,} \quad \alpha n=\frac{4}{3} \tag{17}$$

So the acceleration demands $n<\frac{4}{3}$. Again from equation (13) the weak energy condition ($\rho>0$) demands

$$2\alpha-\frac{2\omega}{3}-\frac{5}{3}>0$$

This implies that $\omega<3\alpha-\frac{5}{2}$. From equation (15) and (13) it is clear the perfect fluid follows an equation of state of the form $p=\gamma_m\rho$, where the index $\gamma_m$, given by

$$\gamma_m=\frac{2-\beta}{3\alpha}-1, \tag{18}$$

where $\gamma_m$ lies within the interval $0<\gamma_m<1$. This restricts $\alpha$ within the range $\frac{4}{9}<\alpha<\frac{8}{9}$. Infact for present matter dominated universe ($\gamma_m=0$), $\alpha=\frac{8}{9}$. But this does not satisfy the criteria for acceleration ($\alpha>1$) and hence the universe decelerates with a perfect fluid CDM ($0\leq\gamma_m<1$) with a potential (2) in BD theory.

Now we consider a CDM which has a dissipative effect and we are particularly interested in a present day universe i.e, $p=0$. Under such condition equation (11) takes the form

$$\dot{\rho}+3\frac{\dot{R}}{R}(\rho+\pi)=0 \tag{19}$$

As is mentioned earlier this type of dissipative effect in FRW cosmology can be modelled in two ways. Generally the dissipative effect is accounted by conventional bulk viscous effect. In FRW universe the bulk viscosity can be modelled within the framework of non-equilibrium thermodynamics proposed by Israel and Stewart [31]. According to this theory the bulk viscous pressure $\pi$ follows the transport equation

$$\pi+\tau\dot{\pi}=-3\eta H-\frac{\tau\pi}{2}[3H+\frac{\dot{\tau}}{\tau}-\frac{\dot{T}}{T}-\frac{\dot{\eta}}{\eta}] \tag{20}$$

where the positive definite quantity $\eta$ stands for the coeffecient of bulk viscosity, $T$ is the temperature of the fluid and $\tau$ is the relaxation time associated with the dissipative effect i.e, the time taken by the system to reach equilibrium state if the dissipative effect is suddenly switched off. Considering the divergence term in the square bracket to be small i.e, $(\frac{R^3\tau}{\eta T})$ to be constant, the equation can be approximated to a simpler form

$$\pi+\tau\dot{\pi}=-3\eta H \tag{21}$$

In literature this is commonly described as a truncated version of the full nonequilibrium thermodynamics. The viscous effects are assumed to be not so large as observation seems to rule out huge entropy production in large scales [32]. Usually $\tau$ is expressed as $\frac{\eta}{\rho}$ so as to ensure that the viscous signal does not exceed the speed of light [33] and also $(\tau H)^{-1}=\nu$, where $\nu>1$ for a consistent hydrodynamical description of the fluid [34]. With these two assumption equation (21) becomes

$$\nu H+\frac{\dot{\pi}}{\pi}=-\frac{3\rho H}{\pi}. \tag{22}$$

In a very recent work Chimento et al [26] have shown that a mixture of minimally coupled self interacting scalar field and a perfect fluid is unable to drive accelerated expansion and solve the cosmic coincidence problem at the same time, while the mixture of a dissipative CDM with bulk viscosity along with minimally coupled self interacting scalar field can successfully drive the accelerated expansion and solve the cosmic coincidence problem simultaneously.

An effective negative pressure can also be created from cosmic antifriction which is closely related to particle production out of gravitational field. In a recent paper Zimdahl *et al* [28] have shown that one can have a negative $\pi$ if there exists a particle number nonconserving interaction inside matter. This may happen due to particle production out of gravitational field. In this case, the matter is of course not a dissipative fluid, but a perfect fluid with varying particle number. Though substantial particle production is an event that occurs in early universe, Zimdahl *et al* have shown that extremely small particle production rate can also cause sufficiently negative $\pi$ to violate strong energy condition.

We donot apriori assume any specific model for this dissipative effect, rather only assume the existence of a negative $\pi$. For a similar kind of evolution of the scale factor and the scalar field given by equation (12), the energy density for the fluid, with negative pressure is also given by equations (13) and (14). From equations (19) and (13), one can easily find that

$$\pi = \frac{(2-\beta-3\alpha)\phi_0}{t_o^{\beta}}\left[\frac{2\alpha+\beta(1+\alpha)-\beta^2(1+\omega)}{2-\beta}\right]t^{\beta-2} \tag{23}$$

From equation (23) one can easliy check that to have a negetive $\pi$, one should have $3\alpha > 2-\beta$ which essentially means $\alpha > \frac{8}{9}$. This suits the condition for acceleration as an $\alpha$ is needed to be greater than 1 for that. One can also check that for this set of solutions given by (12), (23) and (13) and from equations (22), the condition $\nu > 1$ holds provided $2-\beta > 0$, which is very much true in our case. This is important for the hydrodynamical description if the CDM is assumed to be a conventional viscous fluid.

To have a clear picture of the expansion of the universe and the missing energy, we further study the energy density and pressure of the geometric scalar field. The expression for the energy density and the pressure of the scalar field can be derived from the field equations (8) and (9) to be

$$\rho_\phi = \left[\frac{\omega}{2}\,\frac{\dot{\phi}^2}{\phi} + \frac{V}{2} - 3\frac{\dot{R}}{R}\dot{\phi}\right] \tag{24}$$

and

$$p_\phi = \left[\frac{\omega}{2}\,\frac{\dot{\phi}^2}{\phi} - \frac{V}{2} + \ddot{\phi} + 2\frac{\dot{R}}{R}\dot{\phi}\right] \tag{25}$$

In case of power law expansion (12) and potential like (2) the energy density of the BD field becomes

$$\rho_\phi = \frac{\alpha\phi_0}{2t_0^{\beta}}\left\{3\alpha+\omega+\frac{5}{2}\right\}t^{\beta-2} \tag{26}$$

and pressure of the BD field is

$$p_\phi = \left[-\frac{\alpha}{2}\left(3\alpha+\omega+\frac{5}{2}\right)+\frac{2}{3}\left(\alpha+\frac{2}{3}\omega+\frac{5}{3}\right)\right]\frac{\phi_0}{t_0^{\beta}}t^{\beta-2} \tag{27}$$

The positivity condition for the scalar energy density demands $3\alpha+\omega+\frac{5}{2} > 0$ which eventually imposes some restriction on $\omega$ that $\omega > -(3\alpha+\frac{5}{2})$. So essentially the two positivity energy condition limits the range of $\omega$ within $-(3\alpha+\frac{5}{2}) < \omega < 3\alpha-\frac{5}{2}$. Clearly a barotropic relation $(p_\phi = \gamma_\phi\rho_\phi)$ is followed by the scalar field, where the adiabatic index $\gamma_\phi$ is given by

$$\gamma_\phi = \; -1+\frac{\frac{2}{3}\left(\alpha+\frac{2}{3}\omega+\frac{5}{3}\right)}{\frac{\alpha}{2}\left(3\alpha+\omega+\frac{5}{2}\right)} \tag{28}$$

The range of $\gamma_\phi$ that agrees with the observational datas and describes the current acceleration for the universe well, is $-0.6 > \gamma_\phi > -1$. One can adjust the value of $\alpha$ and $\omega$ so as to get the required value of $\gamma_\phi$. We now recast equation (8) in the form

$$\Omega_m + \Omega_\phi = 1 \tag{29}$$

where the density parameters for matter $\Omega_m$ and scalar field $\Omega_\phi$ are defined to be (see ref [35])

$$\Omega_m = \frac{\rho}{3H^2\phi} \quad \text{and} \quad \Omega_\phi = \frac{\rho_\phi}{3H^2\phi} \tag{30}$$

The expression for density parameters at present epoch are

$$\Omega_{m0} = \frac{\rho_0}{3H_0^2\phi_0} = \frac{1}{2} - \frac{1}{6\alpha}(\omega+\frac{5}{2}) \tag{31}$$

and

$$\Omega_{\phi0} = \frac{\rho_{\phi0}}{3H_0^2\phi_0} = \frac{1}{2} + \frac{1}{6\alpha}(\omega+\frac{5}{2}) \tag{32}$$

Like $\gamma_\phi$, the value of $\Omega_{m0}$ that suits best the luminosity distance-redshift data for type Ia supernovae is $\Omega_{m0} = 0.28$ and in a similar fashion like $\gamma_\phi$, one can adjust $\alpha$ and $\omega$ value to get the required value of $\Omega_{m0}$.

A point to note here is that in BD theory the gravitational coupling $G$ varies inversely with the scalar field $\phi$. At present time $\phi$ approaches a constant value $\phi_0$, the inverse of which gives the present newtonian constant $G_N$. In the weak field limit the present newtonian coupling and the asymptotic value of $\phi$ is related by

$$G_N = \frac{2\omega+4}{2\omega+3}\;\frac{1}{\phi_0} \tag{33}$$

We wish to find the range of the parameters $\alpha$ and $\omega$, which suits the permissible range of $\gamma_\phi$ and $\Omega_{m0}$ of the quintessence proposals to tally with observations. In figure 1, we have shown the allowed region in the $(\alpha, \omega)$ parameter space (shaded portion in the figure) for the specified range of $\gamma_\phi$ ($-0.6 > \gamma_\phi > -0.8$) and $\Omega_{m0}$ ($0.5 > \Omega_{m0} > 0.3$). The minimum and maximum possible value for $\alpha$ is $1.11$ and $2.857$ respectively, which certainly satisfies the acceleration criteria. The minimum and maximum possible value for $\omega$ is $-2.5$ and $0.928$ respectively, which definitely obeys the restriction imposed by the postivity energy conditions. The present day variation of the gravitational coupling $G$, is $\frac{\dot{G}}{G}|_0 = \frac{2}{3\alpha}H_0$, where $H_0$ ($=\frac{\alpha}{t_0}$) is the Hubble parameter at present. For this range of $\alpha$ this value is $< 10^{-10}$ per year [36].

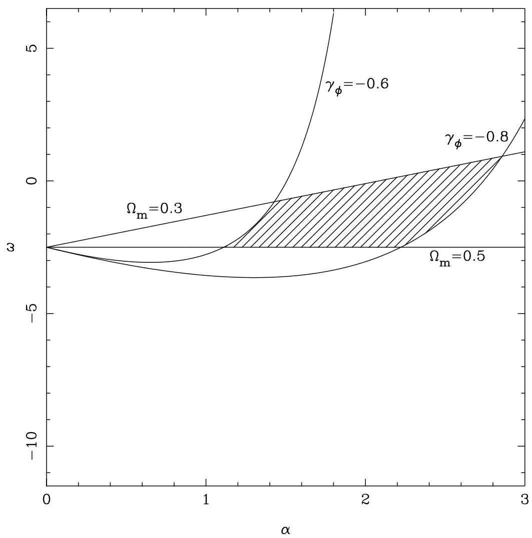


FIG. 1. $\omega$ *vs* $\alpha$ for $-0.6 > \gamma_\phi > -0.8$ and $0.5 > \Omega_{m0} > 0.3$

To analyse the nature of acceleration and our ansatz more critically, it is interesting to match different cosmological parameters with observations. We intend to find the age of the universe and the luminosity distance-redshift relation compatible with our model, probing the background dynamics, that could differentiate between different types of universe.

Since one of the main incentive for reconsidering the introduction of cosmological constant was the age of the universe, we first consider the age of the universe suggested in our model and the constraints imposed on it by observations. Equation (8) can also be presented as

$$H^2 = H_0^2[\Omega_{m0} + \Omega_{\phi 0}](1+z)^{\frac{2}{\alpha}} \tag{34}$$

where $z$ is the redshift defined by

$$1 + z = \frac{R_{observed}}{R_{emitted}} \tag{35}$$

From equation (34) we find the age of the observable universe from a given redshift $z$ is

$$t_0 - t = \frac{\alpha}{H_0(\Omega_{m0} + \Omega_{\phi 0})^{\frac{1}{2}}}\left[1 - \frac{1}{(1+z)^{\frac{1}{\alpha}}}\right] \tag{36}$$

Of course from $t = 0$, i.e, for infinite redshift the age of the universe is $t_0 = \frac{\alpha}{H_0}$.

An old object observed at a certain redshift selects all models with at least that age at that given redshift. In that respect, several age constraints have recently appeared in the literature [37]. For example, the age of the radio galaxy 53W091 observed at a redshift $z = 1.55$ puts a lower bound of 3.5 Gyrs at that redshift. The quasar observed at $z = 3.62$ sets a lower bound of 1.3 Gyrs. In figure 2 we present a plot of the age of the universe as a function of redshift for various values of $\alpha$. Taking into account the range of $\alpha$, prescribed by figyre (1), our universe has an age limit of $15.5Gyrs \leq t_0 \leq 32Gyrs$. From the figure it can be seen that all the universes displayed, besides that for Einstein-desitter one, can accomodate these constraints.

Now we would like to trace the change of luminosity distance

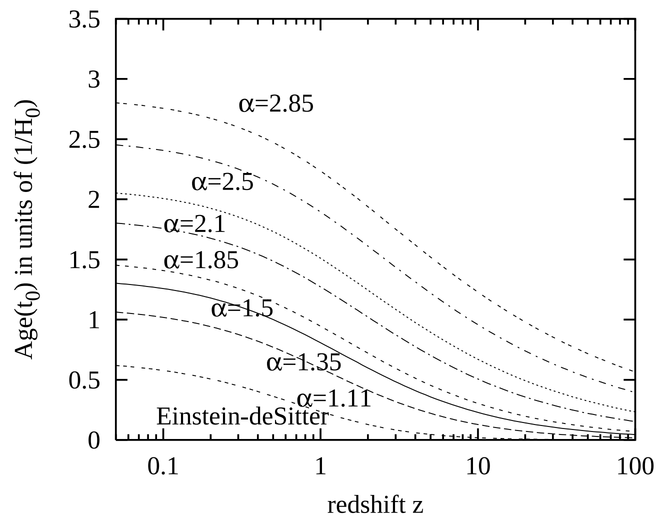


FIG. 2. Age ($t_0$) *vs* redshift $z$ for different $\alpha$ values

with respect to the redshift in our model so as to compare it with the present data available. The result that reveal the so called acceleration of the universe was the observation of the luminosity distance as a function of redshift for type Ia supernovae, which is believed to be a standard candle. From almost 60 redshifts, 42 high redshift data obtained by Supernova Cosmology Project and 18 low redshift observed by Calan Tololo Supernova Survey, favours a universe with positive cosmological constant. Assuming flatness in context of general relativity, the best fit for these data occurs for $\Omega_{m0} = 0.28$ and $\Omega_{\lambda 0} = 0.72$. Optical astronomers measure luminosities in logarithmic units, called magnitudes, given by

$$m(z) = \mathcal{M} + 5\log d_L + 25 \tag{37}$$

where $\mathcal{M}$ is the absolute magnitude and $d_L$ is luminosity distance defined by

$$d_L = R(t_0)(1+z)r_1 \tag{38}$$

for an event at $r = r_1$ at time $t = t_1$. According to our ansatz the expression for $d_L$ is

$$d_L(z) = \frac{(1+z)}{H_0(\Omega_{m0} + \Omega_{\phi 0})^{\frac{1}{2}}}\int_0^z F(z\prime)dz\prime \tag{39}$$

where $F(z) = \frac{1}{(1+z)^{\frac{1}{\alpha}}}$.

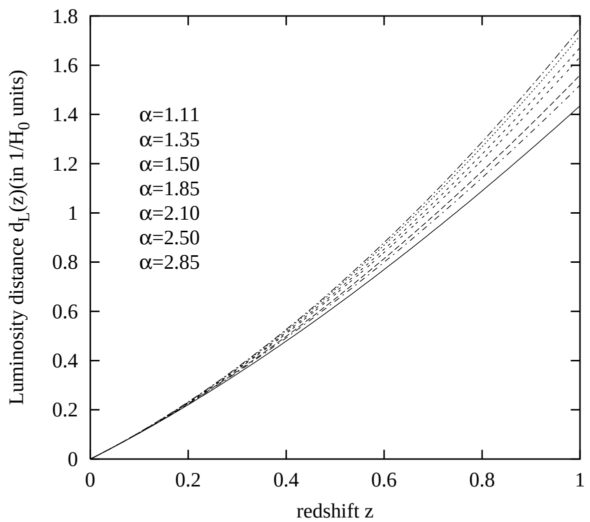


FIG. 3. Luminosity distance $(d_L)$ *vs* redshift $z$ for different $\alpha$ values

In figure 3 we have plotted this luminosity distance versus redshift for different values of $\alpha$. We see that for different $\alpha$ value the $d_L$ is pratically same for lower redshifts upto $z \sim 0.3$. At redshifts $z > 0.3$ the curves becomes separate, but is not distinctly separate to discriminate and rule out different types of the models. Therefore, high accuracy measurements with uncertainties at percentage level are needed in order to cleanly distinguish the models and need to go to redshifts sensibly higher than 1 is evident. In this respect it is very much relevant to mention that Supernova Acceleration Probe (SNAP) is planned to make measurements with an accuracy at percentage level upto redshifts $z \sim 1.7$

## III. DISCUSSION

This work investigates the possibility of getting an accelerated universe in context of BD theory with a symmetry breaking kind of potential and a matter field. In this work we have not used quintessence field to trace the missing energy. The BD scalar field , which is a geometric scalar field, plays the role of dynamical $\Lambda$ and provides that missing energy. It is found that for a power law expansion of the universe, a perfect fluid kind of matter (both pressureless and with pressure) cannot support a late time acceleration of the universe, if the geometric scalar field have potential given by equation (2). While a matter with a dissipative effect can provide the acceleration that agrees with the obervational data sets. The dissipative effect accounted by the negative pressure can be modelled in two ways according to the recent investigations [28,26]. Particle production out of gravitational field can give rise to negative pressure while energy can also be dissipated by bulk viscous effect between the CDM particles. Whatever be the model, the negative pressure generated in either way is sufficient to violate the strong energy condition. For bulk viscous stress, it is assumed that this dissipative effect is not so large as observation rules out huge entropy production in large scales. In this work though no particular model is considered for the origin of the negative pressure, it is found that $\nu > 1$. This is important for hydrodynamical description if the CDM is assumed to be a conventional dissipative fluid. It is assumed that the dissipative effect is sufficiently small. It has been found that matter with a negative stress support a late time accelerated phase of the universe if BD scalar field has symmetry breaking type of potential.

The accelerated solution depends crucially upon two parameters: $\alpha$ and BD parameter $\omega$, both of which are constrained by different physical conditions. Different combinations of $\alpha$ and $\omega$ can produce the required values for $\gamma_\phi$ ($-0.6 > \gamma_\phi > -1$) and $\Omega_m$ ($\sim 0.3$) that matches with present obseravtion that suggest acceleration. We have graphically represented the allowed region of these two parameters for $-0.6 > \gamma_\phi > -0.8$ and $0.5 > \Omega_{m0} > 0.3$. A point to note here is that $\omega$ has small positive as well as negative values. The positivity energy condition is mainly responsible for negative $\omega$. Interestingly the negative value of $\omega$ is supported by the low energy models of string theory and Kaluza Klein theories. [38] To match different cosmological parameters with observation, we calculate the variation of gravitational coupling, age of the universe and the luminosity-distance redshift relation. All of these cosmological parameters agrees quite well with the recent observations.

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[1] S.Perlmutter, M.Della Valle et al., *Nature*, **391**, (1998); S.Perlmutter, G.Aldering, G.Goldhaber et al., *Astroph. J.*, **517** (1999)

[2] P.M. Garnavich, R.P. Kirshner, P. Challis et al., *Astroph . J.*, **493**, (1998); A.G. Riess, A.V. Philipenko, P. Challis et al., *Astron. J.*, **116**, 1009 (1998)

[3] P. Bernadis et al., *Nature*, **404**, 955 (2000)

[4] S.Hanany et al., *astro-ph*/0005123; A. Balbi et al., *astro-ph*/0005124

[5] N.A.Bahcall, J.P.Ostriker, S.Perlmutter and P.J.Steinhardt, *Science*, **284**, 1481 (1988)

[6] V.Sahni and A.Starobinsky, *Int. J. Mod. Phys. D* to appear (2000) *astro-ph*/9904398.

[7] R.R.Caldwell, R.dave and P.J.Steinhardt, *Phys. Rev. Lett*, **80**, 1582 (1998)

[8] P.J.E.Peebles and B.Ratra, *Astrophys.J.Lett.*, **325**, L17, (1988); P.G.Ferreira and M.Joyce, *Phys.Rev.Lett.*, **79**, 4740 (1987); E.J.Copeland, A.R.Liddle and D.Wands, *Phys.Rev.D*, **57**, 4686 (1988)

[9] P.J. Steinhardt, L.Wang and I.Zlatev, *Phys.Rev.Lett.*, **59**, 123504 (1999)

[10] I.Zlatev, L.Wang and P.J.Steinhardt, *Phys.Rev.Lett.*, **82**, 896 (1999)

[11] P.G.Ferreira and M.Joyce, *Phys.Rev.D*, **58**, 023503 (1998); B. Ratra and P.J.E. Peebles, *Phys. Rev. D*, **37**, 3406 (1988); T.Barreiro, E.J.Copeland and N.J.Nunes *astro-ph*/9910214.

[12] Elisa De Pietro and Jacques Demaret gr-qc/9908071.

[13] R.R.Caldwell astro-ph/9908168.

[14] V.Sahni and L.Wang, *astro-ph*/9910097

[15] L.A.U.Lopez and T.Matos, *astro-ph*/0003364

[16] J.P.Uzan, *Phys.Rev.D*, **59**, 123510 (1999)

[17] T.D. Saini, S. Raychauchaudhury, V. Sahni and A.A. Starobinsky, *Phys.Rev.Lett*, **85**, 1162 (2000)

[18] A.R.Liddle and R.J.Scherrer, *Phys.Rev.D*, **59**, 023509 ((1998)

[19] V.Faraoni; *gr-qc*/0002091

[20] G. Esposito-Farese and D. Polarski, *gr-qc*/0009034

[21] N.Bertolo and M.Pietroni, *Phys.Rev.D*, **61**, 023518 (1999)

[22] O.Bertolami and P.J.Martins, *Phys.Rev.D*, **61**, 064007(2000)

[23] R.Ritis, A.A.Marino, C.Rubano and P.Scudellaro, *Phys. Rev. D*, **62**, 043506 (2000)

[24] S. Sen and T.R. Seshadri, *gr-qc*/0007079

[25] B. Boisseau, G. Esposito-Farese, D. Polarski and A.A. Starobinsky, *Phys. Rev. Lett*, **85**, 2236 (2000)

[26] L.P. Chimento, A.S. Jakubi and D. Pavon, *Phys. Rev. D*, **52**, 063509 (2000)

[27] D.N. Spergel and P.J. Steinhardt, *Phys. Rev. Lett.*, **82**, 896 (1999); J.P. Ostriker, *astro-ph*/9912548; S. Hannestad, *astro-ph*/9912558, B. Moore et al., *astrophys. J.*, **535**, L21 (2000)

[28] W. Zimdahl, D.J. Schwarz, A.B. Balakin and D. Pavon, *astro-ph*/0009353

[29] C.Brans and R.H.Dicke, *phys.Rev.*, **124**, 925 (1961)

[30] J. McDonald, *hep-ph*/0007117

[31] W. Israel and J. M. Stewart, *Phys. Lett.* **A58** 213 (1976).

[32] F.W.Stecker, D.L. Morgan, Jr. and J.Bredekamp *Phys. Rev. Lett*, **27**, 1469 (1971).

[33] V. A. Belinski, E. S. Nikomarov and I. M. Khalatnikov,*Sov. Phys-JETP*, **50**, 213 (1979); D. Pavon, J. Bafauly and D. Jou, *Class. Quantum .Grav.*, **8**, 347 (1991); V. Romano and D. Pavon, *Phys. Rev. D*, **50**, 2572 (1994).

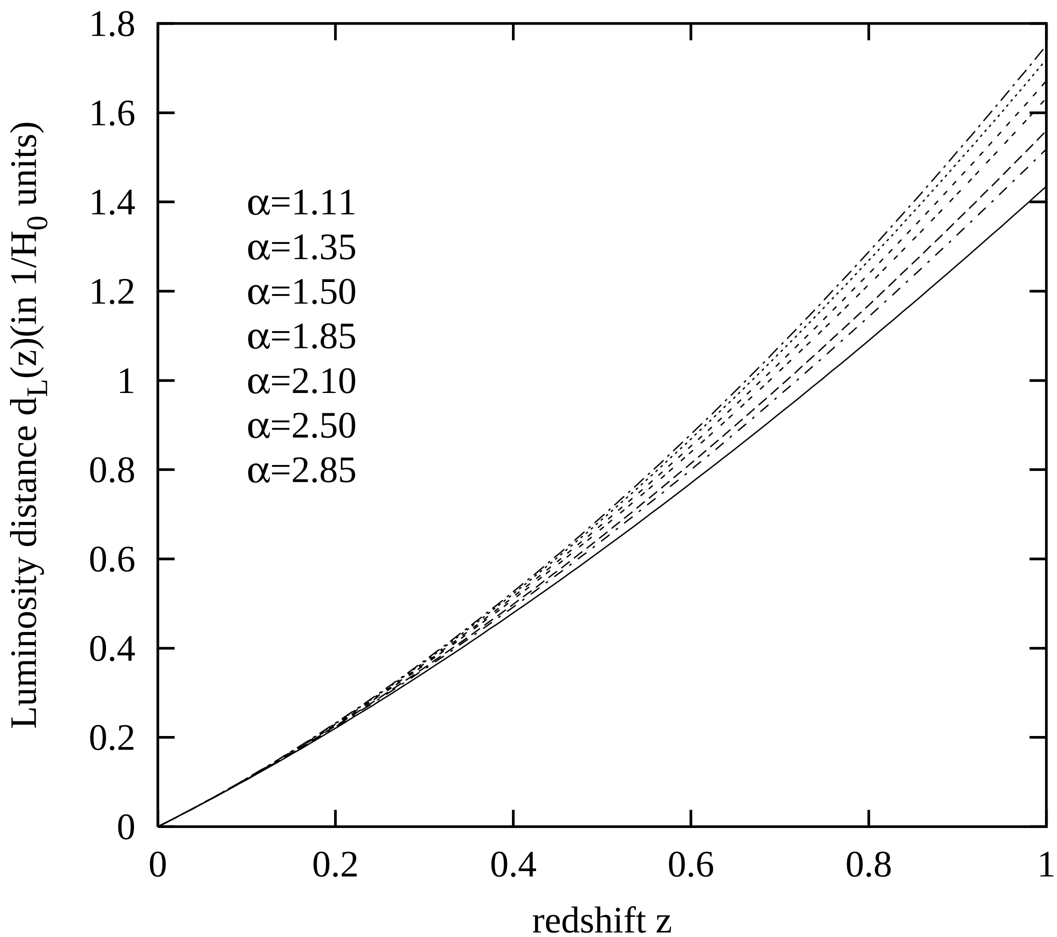
[34] A. A. Coley, R. J. van den Hoogen and R. Maartens, *Phys. Rev. D*, **54**, 1393 (1996).

[35] L.M. Diaz-Rivera and L.O. Pimental, *Phys. Rev. D*, **60**, 123501 (1999)

[36] C.M.Will *Theory and Experiments in Gravitational Physics*, (Rev.Ed. Cambridge University Press, Cambridge, 1993).

[37] J. Dunlop et al., *Nature*, **381**, 581 (1996); Y. Yoshii, T. Tsujimoto and K.Kawara, *Astrophys. J.*, **507**, L113 (1998)

[38] S.J.Kolitch and M.Eardley, *Ann. of Phys*, **241**, 128 (1995)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redshift z
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α=1.85
α=2.10
α=2.50
α=2.85
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
0
0.2
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1